6.G.2 *Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.*

Imagine that we need to find (measure) the volume of rectangular prism to the right (assume all lengths are in inches).

In 5th grade we learned that we can measure the volume of rectangular prisms whose dimensions are whole numbers by filling the prisms with unit cubes. We also saw that we can use the formula V=lwh or V=Bh as a tool to expedite the computation. We don’t know, however, if those formulas work when the lengths are not whole numbers.

We can think of 6.5 as $\frac{13}{2}$ and 8.5 as $\frac{17}{2}$. We can also think of 4 as $\frac{8}{2}$. In other words, every length can be measured in terms of 1/2 inch segments. It seems as though we could use a cube that is ½ x ½ x ½ inches instead of a 1 x 1 x 1 cube to measure the volume.

Look at the figure to the left. The volume of the ½ x ½ x ½ cube is what fraction of the 1 x 1 x 1 cube’s volume?

This seems to support the notion that that the volume, when calculated using fraction lengths, is the product of the length, width, and height: ½ x ½ x ½ = ?

But let’s keep exploring.

In terms of ½” cubes, we can see that we would need 13 x 17 =221 to perfectly fill up the base layer of the prism. Since the height is 4” we would need 8 layers of the base to fill the prism, and 221 x 8 = 1768. It would take 1,768 of these ½” cubes to fill the prism. But we must remember that each of these cubes has a volume measure of 1/8 in3. To report the measure of the volume in terms of cubic inches we need 1/8 of 1768 which is 221. Therefore our volume, measured in cubic inches, is 221 in3.

Does this match what we would also get if we used the formula V=lwh or V=Bh? Let’s check:

$$6.5\*8.5\*4=?$$

 Does this mean there are two different volumes for the prism? Most certainly not. The volume is the space inside the prism. I can measure it using 1” cubes and say the volume is 221 times as large as a 1” cube. I can also measure the volume in using ½” cubes. In that case I would say the volume is 1768 times as large as a ½” cube. I could even measure it using a 12” cube. It would be the same volume, but I would be finding the number of cubic feet that it would take to fill the prism.

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